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# Down Range Observer Location

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MONOGRAPH NR. 1  
DOWNRANGE OBSERVER LOCATION

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### INTRODUCTION:

→ In order to better control the results of the experiments conducted downrange, it is necessary to find, for a particular firing, the optimum location of the ship and aircraft. ~~It is the purpose of this note to give a method for determination of these optimum observing positions,~~ together with a knowledge of the range squared to the target when it is at an altitude of interest, <sup>is presented.</sup> From the information given one may also determine the target steradianance required in order that an instrument of known sensitivity could detect it.

### THEORY:

The primary assumption made in the following is a straight line trajectory. Therefore, the impact point is the so called "vacuum impact." For corrections to real splash point or "atmospheric impact" see

### DISCUSSION.

From Figure 1 - notice that

$$\frac{a}{\tan \theta_e} = b + x \quad (1)$$

$$\text{and } b^2 + y^2 = R'^2 \quad (2)$$

$$\text{and } R'^2 + a^2 = R^2 \quad (3)$$

$$\therefore R^2 = a^2 + y^2 + b^2 \quad (4)$$

$$\text{and from (1)} \quad R^2 = a^2 + y^2 + \left( \frac{a}{\tan \theta_e} - x \right)^2 \quad (5)$$

$$\text{or } R^2 = a^2 + y^2 + \frac{a^2}{\tan^2 \theta_e} - \frac{2ax}{\tan \theta_e} + x^2 \quad (6)$$

$$\text{or } R^2 = a^2 \left( \frac{1 + \tan^2 \theta_e}{\tan^2 \theta_e} \right) + a \left( -\frac{2x}{\tan \theta_e} \right) + y^2 + x^2 \quad (7)$$

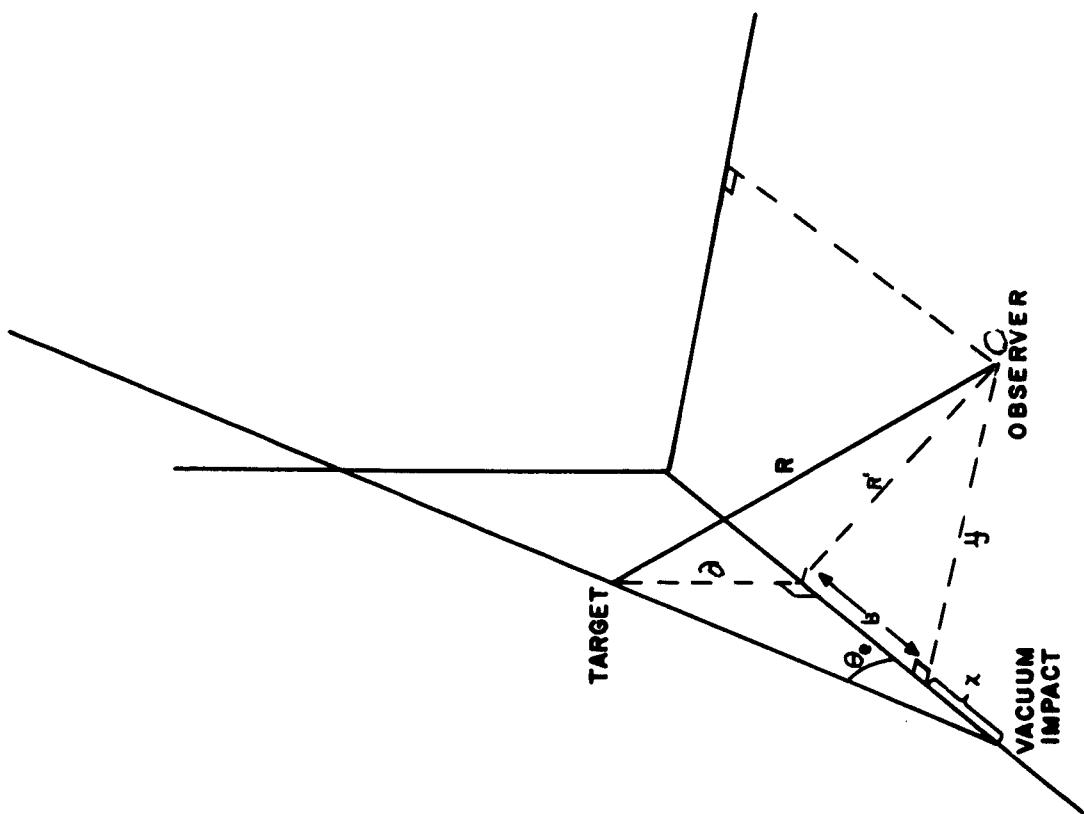


Figure #1

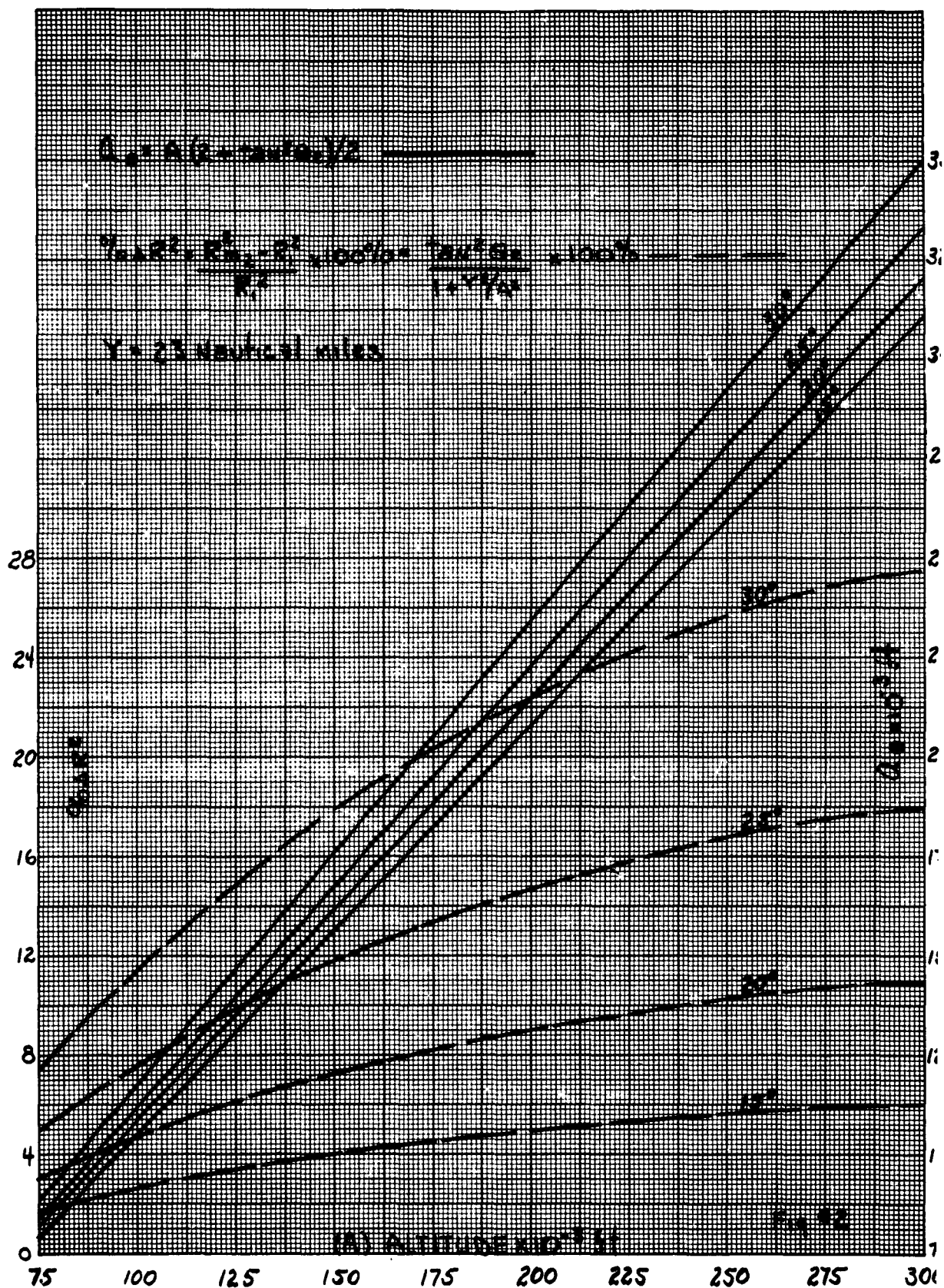
in order to find the optimum values of x and y for observations of a target at a particular altitude  $a = A$ , take  $\left(\frac{\partial R^2}{\partial a}\right)_{a=A}$  and set it equal to zero

$$\left(\frac{\partial R^2}{\partial a}\right)_{a=A} = 2A \frac{(1 + \tan^2 \theta_c)}{\tan^2 \theta_c} - \frac{2x}{\tan \theta_c} = 0$$

$$\text{or } y = A \frac{(1 + \tan^2 \theta_c)}{\tan \theta_c} \quad (9)$$

Another look at Figure 1 indicates immediately that the value of y for minimum  $R^2$  in any case is 0. This indicates that one would like to observe from a position as close to the plan of the trajectory as is commensurate with safety and equipment limitation. The position up range at which one should observe is indicated by equation (9). It will be noted that this position does not correspond to a location directly under the target at the altitude of interest A which would be simply  $y = A / \tan \theta$  but is farther up range. In fact it represents the position for which the line of sight to the target is perpendicular to the trajectory. This position was chosen, in spite of the fact that the range is larger, because this position favors observations of the target at most altitudes greater than the altitude of interest. The advantages and disadvantages of this choice of location are illustrated in Fig. 2. From that figure it is apparent that for typical re-entry angles the percentage increase in range<sup>2</sup> suffered is small, and also that the altitude above which the range to the target is smaller from the up range





position is only slightly larger than the altitude of interest. Thus for all higher altitudes it is preferable to be in the location described in equation (9). It is important to favor higher altitudes because of the reduced radiance at these altitudes. For convenience equation (9) has been put in nomogram form in Nom. 1. The units which are most convenient for this particular application are indicated on the nomogram.

Equation (9) gives the up range position at which the observer should locate himself, but the question arises what indeed will be the range squared when the target is at altitude A. In order to find that value of  $R^2_{\min}$  substitute (9) into (7) and get

$$R^2_m = \left\{ \frac{A^2(1+\tan^2\theta_e)}{\tan^2\theta_e} - 2A \frac{(1+\tan^2\theta_e)}{\tan^2\theta_e} + \frac{A^2(1+\tan^2\theta_e)^2}{\tan^2\theta_e} + y^2 \right\} \quad (10)$$

Which reduces to

$$R^2_m = A^2(1+\tan^2\theta_e) + y^2 \quad (11)$$

For convenience this equation has been nomogrammed and is found on Nom. 2.

Because equation (11) only gives a value for  $R^2$  at the particular altitude of interest A, for which position was optimized, the question of how much this range squared changes with the altitude of the missile, a, for the position of the observer already found arises. This could be found by plotting curves of equation (7), e.g.,  $R^2$  vs a, but it would require a set of curves for each location. If one is willing to settle for knowing

what the change in altitude of the target will be if the range squared is doubled then let  $R^2$  from equation (7) be equal to  $2R_{\min}^2$  from equation (11) and compare  $a$  to  $A$ .

$$a^2 \left( \frac{1 + \tan^2 \Theta_e}{\tan \Theta_e} \right) + a \left( \frac{-2y}{\tan \Theta_e} \right) + y^2 + x^2 = 2A^2 (1 + \tan^2 \Theta_e) + 2y^2 \quad (12)$$

which is quadratic in  $a$ . When solved for  $a$  (12) gives

$$\Delta a = \pm \tan \Theta_e \sqrt{A^2 + \frac{y^2}{1 + \tan^2 \Theta_e}} \quad (13)$$

this equation gives the variation in altitude  $\Delta a$  for a variation in range squared equal to 2. It is fortunate that this result can be further generalized as follows. If we now label variations in altitude for a factor of  $n$  change in  $R^2$  a new symbol  $\Delta a_n$  then one can write

$$\Delta a_n = \sqrt{n-1} \Delta a_1 \quad (14)$$

Or more practically

$$\Delta a_2 = 2 \Delta a_1 \quad (15)$$

$$\Delta a_{10} = 3 \Delta a_1 \quad (16)$$

For convenience equation (13) has been nomogrammed and is found in Nom. 3.

#### USE OF NOMOGRAM:

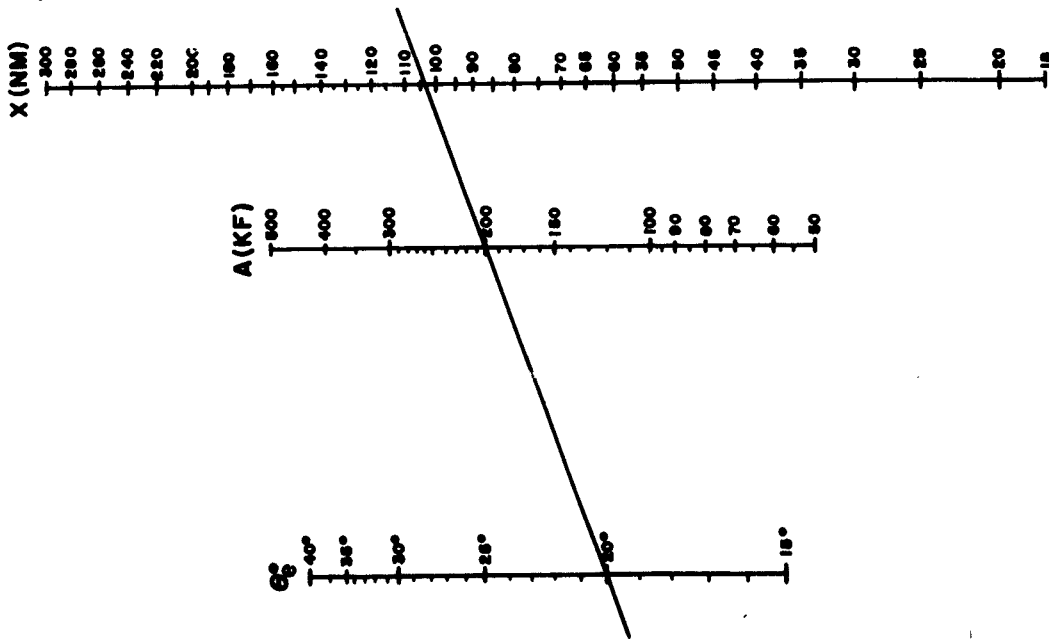
##### 1. Application to Ship:

The optimum position of the ship can be found using Nomogram 1.

The information needed to find this position is the expected re-entry angle in degrees and the "altitude of interest" in Kft. Place a straight edge connecting  $\Theta_e$  and  $A$  and read directly  $X$ , the up range position, in nautical miles. For example if  $\Theta_e = 20^\circ$  and  $A = 200$  Kft. then  $X \approx 103$  NM up range. Let us suppose that  $Y$ , the perpendicular distance from the plan of the trajectory, is 20 NM.

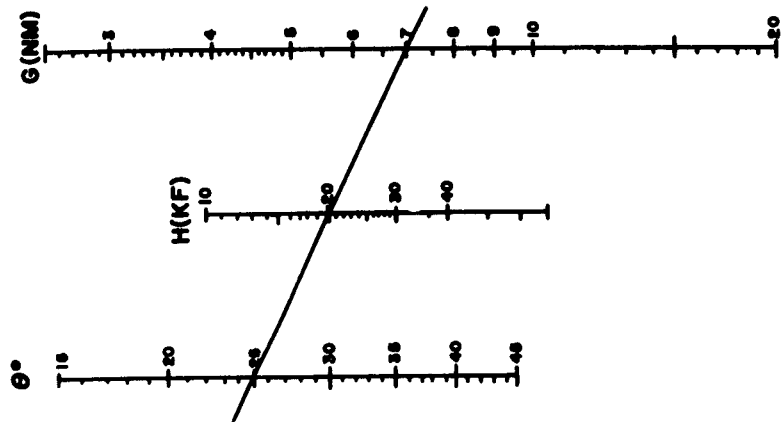
To find the  $R_m^2$  in  $\text{cm}^2$  now nomogram #2 can be used. Using the same  $\Theta_e$  and  $A$  as before connect with a straight edge  $20^\circ$  and 200 Kft and read the  $l_1$  scale. Transfer the reading on  $l_1$  to the same reading

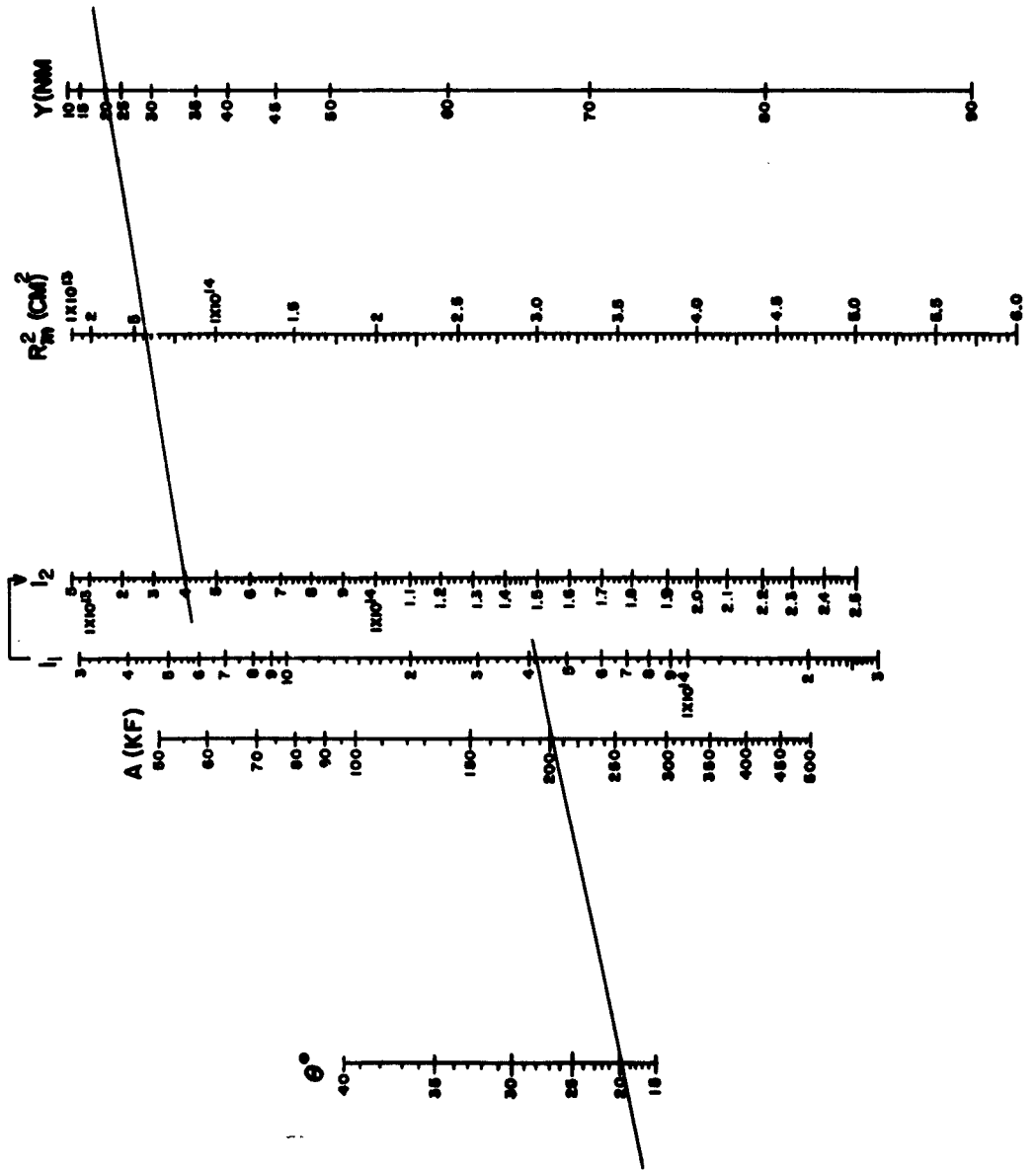
$$X = A \left( \frac{1 + \tan^2 \theta_p}{\tan \theta_0} \right)$$



Use as is for Ship

For Aircraft  
 if altitude of aircraft =  $H$   
 and if altitude of interest =  $A'$   
 take  $A = (A' - H)$   
 and take  $(X / G) = X'$   
 where  $X'$  = Up range position  
 and  $G$  from below.





$$R_m^2 = A^2 (1 + \tan^2 \theta) + 1$$

Use as is for Ship

For Aircraft

if altitude of aircraft = H

and if altitude of interest = A'

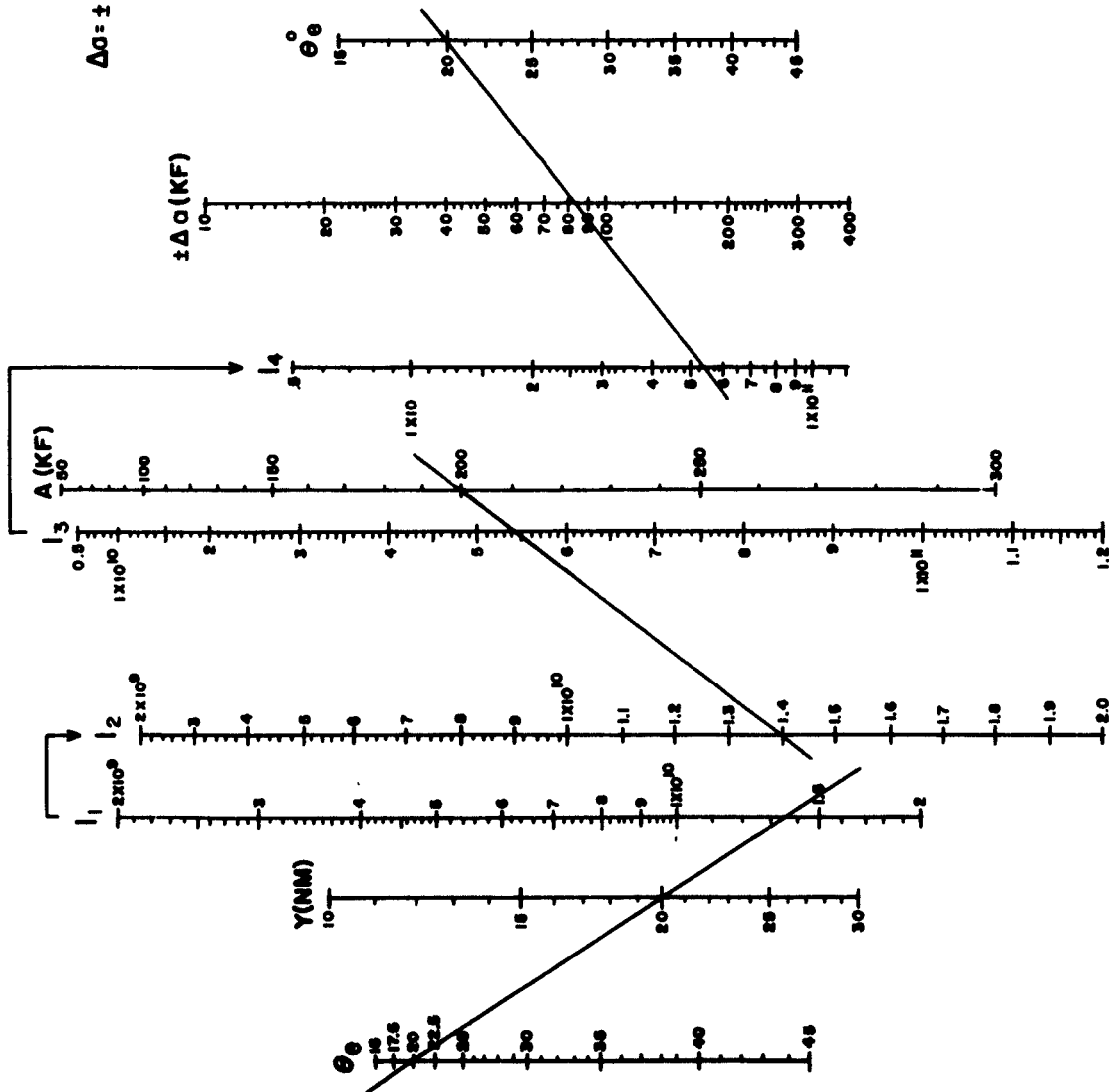
take  $A = (A' - H)$

$$\Delta\theta = \pm \tan \theta_0 \sqrt{A^2 + Y^2}$$

Use as is for Ship

For Aircraft

if altitude of aircraft = H  
and if altitude of interest = A',  
take  $A = (A' - H)$



on  $l_2$  and connect that with the Y which has previously been chosen in the example. Read  $R_m^2$  is  $\text{cm}^2$  which in this case is seen to be  $5.1 \times 10^{13} \text{ cm}^2$ .

Nomogram #3 is used to find  $\Delta a$  in Kft. Again  $\theta_e$ , A and Y must be given. Let us continue the same example viz  $\theta_e = 20^\circ$ , A = 200 Kft and Y = 20 NM. First with a straight edge connect  $\theta_e$  and Y and read the value on  $l_1$  which in this case is  $\pm 5.4 \times 10^{10}$ . Place this value on  $l_4$  and connect it with  $20^\circ$  on  $\theta_e$  and read  $\Delta a$  which is in this case  $\pm 85 \text{ Kft}$ . Therefore, if the range squared is allowed to change by a factor of 2 from  $R_m^2$  the missile will be at altitudes 200 Kft  $\pm 85 \text{ Kft}$  or between 285 Kft and 115 Kft.

## 2. Application to Aircraft:

With minor modification these 3 nomograms calculated for ship position optimization can be used for location of an aircraft. The modifications are as follows:

a. If the altitude of interest for measurement is really A' and the altitude of the aircraft is H then to use the nomogram first take A to be  $A = (A' - H)$  and use A on all three nomograms as one would for the ship.

b. In addition the answer one gets for X in Nom. 1 must be altered. To do this first use the small nomogram on the right in Nom. 1 to find G by connecting  $\theta_e^0$  and H by a straight edge. For example if  $\theta_e^0 = 25^\circ$  and H = 20 Kft then G is  $\pm 7 \text{ NM}$ . Now take  $X' = X \div G$  and this is the up range position of the Aircraft X'.

## DISCUSSION:

### 1. Correction to Atmospheric Impact:

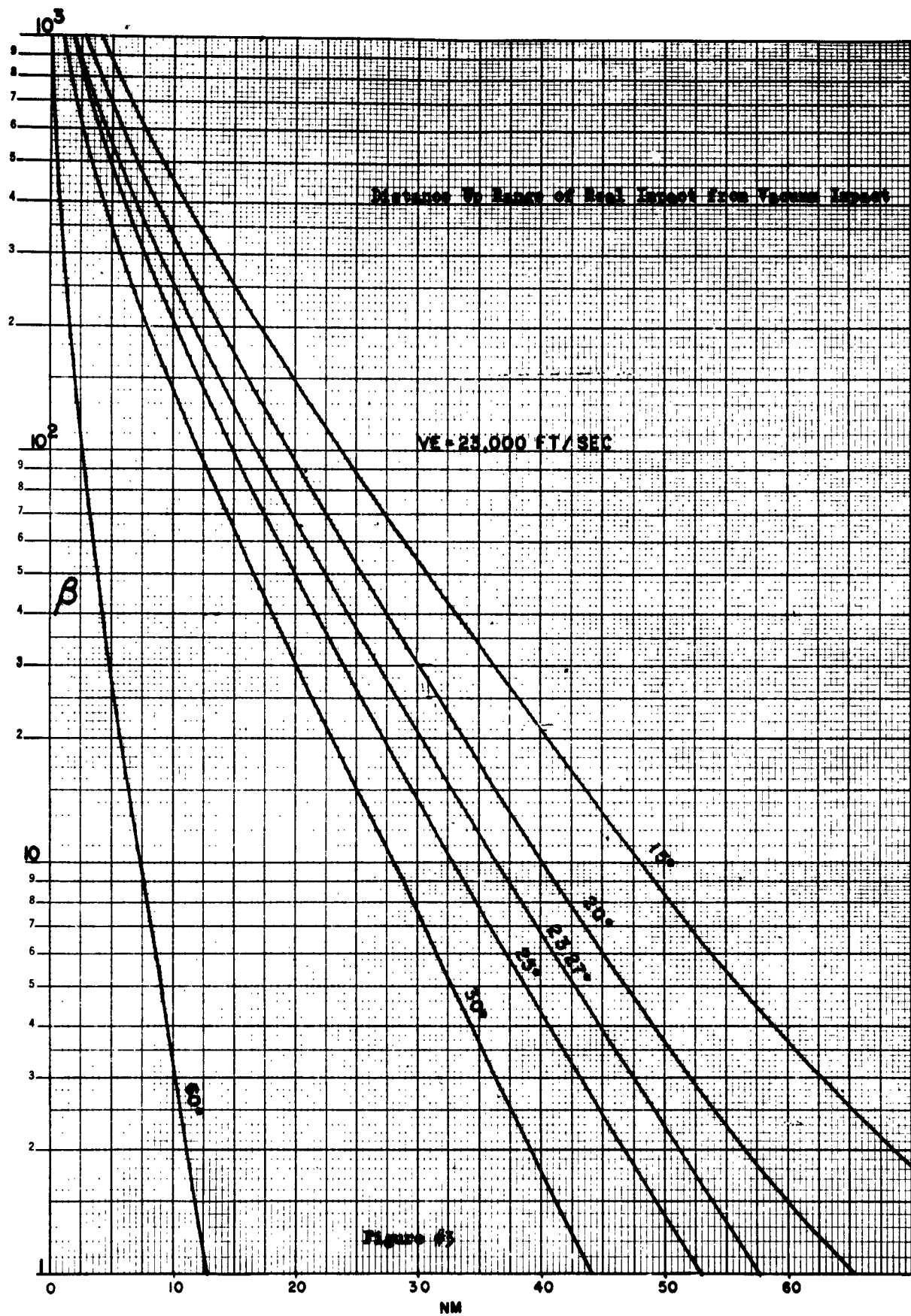
The nomograms described above are calculated on the assumption of straight line trajectory and therefore the up range distance given is distance from the so called "vacuum impact." Because, prior to a shot, one is usually given "atmospheric impact" and not "vacuum impact" the answer one gets from Nom. 1 must be slightly altered. In order to do this one must know the expected re-entry angle and the ballistic coefficient of the target. Knowing these one can find the distance between the "vacuum impact" point and the "atmospheric impact" point from Figure 3. For example for a  $20^\circ$  re-entry body of ballistic coefficient of 100 that distance is 19.5 NM. Therefore, the answer for X found in the example in Use of Nomogram which was 103 NM is really too high by 19.5 NM. Therefore, the up range distance in this example from "atmospheric Impact" should be 83.5 NM. N. B. for very high ballistic coefficients this correction is rather small, although probably not negligible.

### 2. Determination of Threshold Steradiances:

Finding Range squared is not wholly academic. Let the steradiances of the target in the wavelength region of interest be  $J$  in watts/steradian. Let the sensitivity of an instrument in this region be  $S$  in watts/cm<sup>2</sup> (NEPD). Then if the target is  $R$  centimeters away, the minimum steradiances that the target can have and still be "seen" by the instrument is  $J = SR^2$ .

If one then finds  $R_m^2$  from Nom. 2 and knows the NEPD of his instrument he can find how intense the radiation from the target must





be in order to "see it". For example the  $R^2_m$  found in the example in Use of the Nomogram was  $5.1 \times 10^{13} \text{ cm}^2$ . Let us assume that there is an instrument in the lead sulfide region with NEPD =  $10^{-12} \text{ w/cm}^2$  then  $J = 5.1 \times 10^{13} \times 10^{-12} = 51 \text{ watts/ster}$  i.e. this instrument will "see" a target at 200 Kft altitude if it has a steradiance of 51 watts/ster or more.

Furthermore if  $R^2$  changes by a factor of 2 so does J, and since  $\Delta a$  was found to be  $\sqrt{2}$  85 Kft, the instrument will "see" the target at an altitude of 285 Kft if it has a steradiance of 102 watts/ster at that altitude. Further, because  $\Delta a_5 = 2 \Delta a_2$  the instrument will see a target of 255 watts/ster (which is 5 times J for  $R^2_m$ ) at an altitude of 370 Kft.

NOTE: These values of  $-J$  are only the steradiance in the wavelength interval of the instrument, not the total power emitted in all wavelengths.

<u>SYMBOL</u>	<u>DEFINITION</u>	<u>UNITS</u>
a	Altitude of the target	Kft.
A	Altitude of the target for which position is to be optimized. "Altitude of interest" on a particular shot.	Kft.
$\Delta a$	Change in altitude for a 3db change in $R^2$ .	Kft.
X	Uprange position from vacuum impact.	NM
x	Vacuum Impact	Kft.
Y	Cross range position.	NM
y	Cross range position	Kft.
$R_m^2$	Range to target when target is at a = A.	cm <sup>2</sup>
$\theta_e$	Re-entry Angle.	Degrees